Exercise 7

The height (in meters) of a projectile shot vertically upward from a point 2 m above ground level with an initial velocity of 24.5 m/s is $h = 2 + 24.5t - 4.9t^2$ after t seconds.

(a) Find the velocity after 2 s and after 4 s.

(b) When does the projectile reach its maximum height?

(c) What is the maximum height?

(d) When does it hit the ground?

(e) With what velocity does it hit the ground?

Solution

Part (a)

To determine the velocity, take the derivative of the position function.

$$\begin{aligned} v(t) &= \frac{dh}{dt} \\ &= \frac{d}{dt}(2 + 24.5t - 4.9t^2) \\ &= 24.5 - 9.8t \end{aligned}$$

As a result, the velocity after 2 s and after 4 s are, respectively,

$$v(2) = 24.5 - 9.8(2) = 4.9 \ \frac{\text{m}}{\text{s}}$$

$$v(4) = 24.5 - 9.8(4) = -14.7 \ \frac{\text{m}}{\text{s}}.$$

Part (b)

The projectile reaches its maximum height when it comes to a standstill in the air, so set $v(t) = 0$ and solve the equation for t.

$$\begin{aligned} v(t) &= 0 \\ 24.5 - 9.8t &= 0 \\ t &= \frac{24.5}{9.8} \\ t &= 2.5 \text{ s} \end{aligned}$$

Part (c)

To determine the maximum height, plug the time found in part (b) into the position function.

$$h = 2 + 24.5(2.5) - 4.9(2.5)^2$$

$$= 32.625 \text{ m}$$

Part (d)

To determine when the projectile hits the ground, set $h(t) = 0$ and solve the equation for t.

$$h(t) = 0$$

$$2 + 24.5t - 4.9t^2 = 0$$

$$4.9t^2 - 24.5t - 2 = 0$$

$$t = \frac{24.5 \pm \sqrt{24.5^2 - 4(4.9)(-2)}}{2(4.9)}$$

$$t \approx \{-0.0803417, 5.08034\}$$

Choose the positive time, since the launch occurs at $t = 0$ and the landing happens after that.

$$t \approx 5.08034 \text{ s}$$

Part (e)

To determine the velocity when it hits the ground, plug the time found in part (d) into the velocity function.

$$v(5.08034) \approx 24.5 - 9.8(5.08034)$$

$$\approx -25.2873 \ \frac{\text{m}}{\text{s}}$$